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ON EMPIRICAL MODELS OF THE UPPER ATMOSPHERE IN THE POLAR REGIONS

PETER W. BLUM ISADORE HARRIS



SEPTEMBER 1971



GODDARD SPACE FLIGHT CENTER GREENBELT, MARYLAND

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ABSTRACT

The expression for the exospheric temperature in Jacchia's static diffusion models of the upper atmosphere has a discontinuous gradient at the poles. Therefore it cannot describe the true state of the upper atmosphere in the polar regions. Furthermore, it cannot be used to calculate quantities that depend on the derivative of the exospheric temperature, or the density, like pressure gradients, horizontal forces or horizontal heat fluxes. A modified expression for the exospheric temperature is suggested. This modification yields variables of state of the upper atmosphere that deviate little from Jacchia's values, but it has continuous gradient at the poles and is therefore more suitable for treating dynamical problems like the global wind pattern.

ON EMPIRICAL MODELS OF THE UPPER ATMOSPHERE IN THE POLAR REGIONS

INTRODUCTION

In the past decade and a half a very great number of observations of the physical state of the upper atmosphere were obtained both by satellite drag analysis and by direct measurements with rockets and satellites. The analysis of these data led to the development of Jacchia's semi-empirical static diffusion models of the upper atmosphere (Jacchia, 1965 and 1971). These models describe the observed variations of the upper atmosphere generally with great accuracy. In Jacchia's description of the upper atmosphere all state variables depend upon the exospheric temperature, i.e., once the exospheric temperature is determined, the state of the upper atmosphere at any height can be ascertained.

According to Jacchia the exospheric temperature depends upon the solar activity F, the average solar activity \overline{F} during several solar rotation periods, the geomagnetic activity index K_p , the day in the year (semi-annual effect), the local solar time (diurnal variation) and the latitude. The dependence on LST (local solar time) can also be expressed as an azimuthal dependence, where the azimuthal angle ϕ is measured from the azimuth that corresponds to a fixed LST.

The variation of exospheric temperature with local time and with latitude can be given by a single mathematical expression, i.e., the distribution of exospheric temperature on the globe, where the longitude is measured with respect to local noon.

In Jacchia's model the exospheric temperature is given (in his notation) by:

$$T_{\infty} = T_{c} (1 + R \sin^{m} \theta) \left(1 + R \frac{\cos^{m} \eta - \sin^{m} \theta}{1 + R \sin^{m} \theta} \cos^{3} \left(\frac{\tau}{2} \right) \right)$$
 (1)

where T_c depends only upon solar activity and

$$\tau = H + \beta + p \sin(H + \gamma) \qquad (-\pi \le \tau \le \pi)$$

$$\eta = \frac{|\phi - \delta|}{2}$$

$$\theta = \frac{|\phi + \delta|}{2}$$

 ϕ - latitude

 δ - declination of the Sun

H - LST counted from local noon

 $\beta = -37^{\circ}$

 $p = 6^{\circ}$

m = 2.2

R = 0.3

 $\gamma = 43^{\circ}$

For the problem treated in this paper we shall use the notation:

$$T_{\infty} = F_1(\vartheta) + F_2(\vartheta) \cdot F_3(\varphi)$$
 (2)

where ϑ and φ are the colatitude and the longitude respectively, and

$$F_1(\vartheta) = T_c(1 + R \sin^m \theta)$$
 (3)

$$F_2(\vartheta) = T_c R(\cos^m \eta - \sin^m \theta)$$
 (4)

$$F_3(\varphi) = \cos^3\left(\frac{\tau}{2}\right) \tag{5}$$

The subject of this paper is to show that expression (1) for the exospheric temperature cannot describe the physical conditions near the poles in a true fashion. We shall suggest a modification of (1) that deviates very little from it at all latitudes, but will give more consistent results in the polar regions.

JACCHIA'S EXOSPHERIC TEMPERATURE FOR THE POLAR REGIONS

Jacchia's model is a static model; this implies that it is a description of the steady state of the atmosphere. The global variations of temperature and density that result from it should therefore be continuous functions of the independent variables like height, latitude and local time. In nature there appear no discontinuities of the variables of state or their derivatives in the steady state. As in Jacchia's model all variables of state depend upon the exospheric temperature, a discontinuity of it, or of its derivative, will cause also discontinuities of the other variables or their derivatives. We shall show in the appendix that expression (1) has a discontinuous derivative at the poles and therefore cannot be a very good approximation to the state of the upper atmosphere in the polar regions, although a direct comparison of Jacchia's model with observations in the polar regions is difficult, as very few exact observations exist for these regions.

By a derivative of a variable defined on a sphere we understand the derivative of this variable along any arc of this sphere. The value of this derivative depends on the particular arc chosen. By a continuous derivative of the variable at a point of the sphere we mean that the derivative of the variable along an arbitrary arc is continuous.

The discontinuity of the derivative of the expression for the exospheric temperature at the poles can easily be demonstrated by formal differentiation. In this paper we will not do this, but shall analyse (in the appendix) expression (1) in terms of spherical harmonic functions, as our suggested modification becomes more transparent by this method.

All single valued functions that are defined for all points of a sphere given by colatitude ϑ and azimuth ϕ ($0 \le \vartheta \le \pi$, $0 \le \phi \le 2\pi$) that are continuous and have continuous derivatives can be developed into a series of spherical harmonic functions. This series will converge uniformly for all points of the sphere to the value of the function and may be written

$$F(\vartheta,\varphi) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(a_{mn} P_n^m(\cos\vartheta) \cos m\varphi + b_{mn} P_n^m(\cos\vartheta) \sin m\varphi \right)$$
 (5)

Jacchia's function cannot be developed into such a series for all points of the sphere. It has a discontinuous derivative at the pole, or what is equivalent, a cusp at the poles. This cusp can easily be removed by modifying the term $\cos^3(\tau/2)$ that appears in the original expression. By making the difference between the modified term and the original term small, the resulting exospheric temperature will deviate only little from Jacchia's original values.

COLL KALDER

In order to reach this we suggest to substitute for $\cos^3(\tau/2)$ the expression

$$S(\vartheta, \varphi) = \sin^2 \vartheta \cdot \left(\frac{1 + \cos (\varphi - 37^\circ)}{2}\right) + \cos^2 \vartheta \cos^3 \left(\frac{\tau}{2}\right)$$
 (6)

At the equator both expressions are exactly identical. At the poles the function F_2 (ϑ) is equal to zero, so (6) does not influence the value of the exospheric temperature at all. On the other hand the derivatives of the exospheric temperature become continuous by this substitution.

Our suggested expression for the exospheric temperature will be in Jacchia's notation

$$T_{\infty} = T_{c}(1 + R \sin^{m}\theta) \left[1 + R \frac{(\cos^{m}\eta - \sin^{m}\theta)}{1 + R \sin^{m}\theta} \right]$$

$$\left(\cos^2\phi\left(\frac{1+\cos\left(\phi-\frac{37^\circ}{2}\right)}{2}\right)+\sin^2\phi\cos^3\left(\frac{\tau}{2}\right)\right)\right]$$

or in our notation

$$T_{\infty} = F_1(\vartheta) + F_2(\vartheta)S(\vartheta,\varphi)$$

The physical meaning of our modification compared to Jacchia's original expression is to leave the diurnal variation unchanged at the equator. We retain the first order Fourier coefficients at all latitudes, but reduce the zero and the higher order Fourier coefficients gradually to zero as the latitude approaches the pole.

Furthermore, the substitution will not cause any shift in the position of the global maxima and minima of the exospheric temperature, i.e., the modified function will retain the property that the maximum of the temperature coincides in latitude with the subsolar point and shows a phase shift of 37° in longitude. This is easily seen as at the extrema $\cos^3(\tau/2)$ becomes either zero or one and $\cos(\phi-37^\circ)$ either 1 or -1. Substituting these values in (6), the expression becomes independent of latitude and therefore does not influence the derivative of T_∞ with respect to latitude at the longitude of global extrema. The position of the extrema is therefore unchanged.

Tables 1 and 2 show the ratio of the modified temperatures to Jacchia's original values. Nowhere is the deviation greater than 1.7%. The exact shape of the diurnal variation is only changed very slightly, less than can be verified by observations.

Figures 1 and 2 show the lines of equal exospheric temperatures in the polar region. It is immediately apparent that the line that passes through the pole according to the original model has a sharp edge there.

ATMOSPHERIC FORCES

The horizontal driving force in the atmosphere is given by

$$F_b = -(grad_b p) / density,$$

where grad, is the horizontal gradient.

We have calculated the driving forces according to Jacchia's model and according to our modification. In the first case we have found that the driving force jumps at the poles by a factor of 3.5 and at some longitude even shows a change in sign. Our model results in smooth forces at the poles.

In calculating the global wind field it is essential to use a well defined smooth force everywhere on the globe, as discontinuous forces at the poles cause the calculated results to be distorted at all latitude, because the boundary condition can influence the values in the whole domain.

RESULTS

- (1) Jacchia's semi-empiric model of the upper atmosphere does not give a consistent description of the state of the atmosphere in the polar region.
- (2) Especially, the model is unsuitable to calculate pressure forces, heat fluxes and gradients of variables of state in the polar region.
- (3) A modification of Jacchia's formula for the exospheric temperature distribution will make the model smooth in the polar regions and will deviate only slightly from the exospheric temperatures given in Jacchia's original expression at any latitude.

(4) The horizontal pressure forces derived from the modified model deviate considerably from the forces derived from Jacchia's model. At high latitudes the difference in amplitude can amount to 30% of the global maximum force.

APPENDIX

A single valued function $F(\vartheta, \varphi)$ that is continuous and has continuous derivatives at all points of a sphere can be developed into a uniformly convergent series of spherical harmonic functions:

$$F(\vartheta,\varphi) = \sum_{n=0}^{\infty} \sum_{m=n}^{\infty} \left[a_{mn} P_n^m(\cos\vartheta) \cos(m\varphi) + b_{mn} P_n^m(\cos\vartheta) \sin(m\varphi) \right]$$
 (1a)

or as re-arranging terms of a uniformly convergent series is allowed:

$$\begin{split} F(\vartheta,\phi) &= \sum_{m=0}^{\infty} \left[\sum_{n=m}^{\infty} a_{mn} P_{n}^{m} (\cos \vartheta) \cos (m\phi) + b_{mn} P_{n}^{m} (\cos \vartheta) \sin (m\phi) \right] \\ &= \sum_{m=0}^{\infty} \left[A_{m} (\cos \vartheta) \cos (m\phi) + B_{m} (\cos \vartheta) \sin (m\phi) \right] \end{split} \tag{2a}$$

where B_0 (cos ϑ) is defined as identical zero.

We shall use the notation

$$P_n^{m'}(\cos\vartheta) = \frac{d}{d(\cos\vartheta)} P_n^{m}(\cos\vartheta).$$
 (3a)

The spherical harmonic functions have at the poles the following properties:

$$P_n^o \text{ (pole)} = (\pm 1)^n; \quad P_n^m \text{ (pole)} = 0 \text{ if } m \neq 0$$
 (4a)

$$P_n^{m'} \text{ (pole)} = 0 \text{ if } m \neq 1$$
 (5a)

therefore:

$$A_m \text{ (pole)} = B_m \text{ (pole)} = 0, \text{ if } m \neq 0,$$
 (6a)

$$A'_{m}$$
 (pole) = B'_{m} (pole) = 0, if m \neq 1, (7a)

As examples of functions on the sphere that have discontinuous derivatives we may take $\sin \vartheta$ and $\sin \vartheta$ $\cos 2\varphi$. For the first function $A_0 = \sin \vartheta$ and therefore $A_0' = \cos \vartheta$ which is not equal to zero. In the second case the same argument applies to A_2' . This shows that a development of these functions into a uniformly convergent series of spherical harmonics is not possible, they cannot belong therefore to the class of functions that are continuous and have continuous derivatives for all points of the sphere. In fact, these functions have a cusp at the poles.

We shall show that also Jacchia's function is of this type, it is continuous at the poles, but has a cusp there.

In our notation

$$T = F_1(\vartheta) + F_2(\vartheta) \cdot F_3(\varphi)$$
 (9a)

By a numerical analysis of $F_3(\varphi) = \cos^3(\tau/2)$ we obtain:

$$F_3(\varphi) = 0.4194 + 0.4088 \cos(\varphi) + 0.2984 \sin(\varphi) + U(\varphi)$$
 (10a)

where U includes all the higher terms. Their coefficients are decreasing to zero like n^{-4} . The first order terms may also be combined

$$0.4088 \cos (\varphi) + 0.2984 \sin (\varphi) = 0.5061 \cos (\varphi - 37^{\circ})$$
 (11a)

Therefore

$$T_{\infty} = F_{1}(\vartheta) + F_{2}(\vartheta) \cdot 0.4194 + F_{2}(\vartheta) \cdot 0.5061 \cos (\varphi - 37^{\circ})$$

$$(12a)$$

$$+ F_{2}(\vartheta) \cdot U(\varphi)$$

We can now identify:

$$A_0(\cos\vartheta) = F_1(\vartheta) + 0.4194 F_2(\vartheta)$$

$$A_1(\cos\vartheta) = 0.4088 F_2(\vartheta)$$

$$B_1(\cos\vartheta) = 0.2984 F_2(\vartheta)$$
(13a)

while the A_i and B_i with i greater than 1 have the same ϑ -dependence as A_1 . At the poles they behave like F_2 , which means that they are zero, but have a derivative that is not zero.

Simple differentiation of A_0 with respect to ϑ gives the result that its derivative is unequal to zero at the poles. The same is true for all the other A_i and B_i .

The cusp at the poles of the function describing the exospheric temperature can be easily removed if we modify the term \cos^3 ($\tau/2$) in such a way that the values of the resulting A_i and B_i will conform at the poles to the required conditions, but the modified term will deviate as little as possible from the original term.

The modification by which this is achieved is given in the main text.

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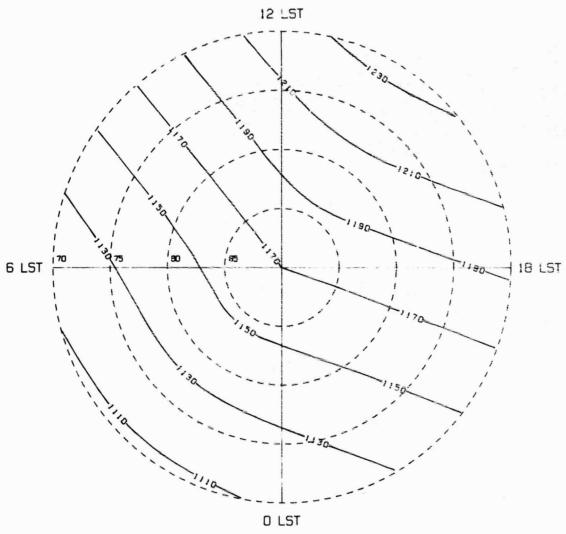
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RATIO OF MONIFIED EXOSPHERIC TEMPERATURE TO JACCHIA'S EXOSPHERIC TEMPERATURE

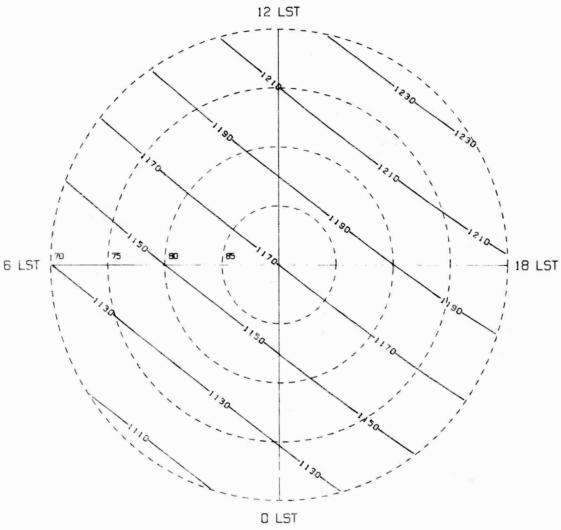
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LINES OF EQUAL EXOSPHERIC TEMPERATURES
FOR POLAR REGION ACCORDING TO JACCHIA MODEL

Figure 1. Lines of Equal Exospheric Temperatures in the Polar Region According to Jacchia's Model for the Global Temperature Distribution



LINES OF EQUAL EXOSPHERIC TEMPERATURES
FOR POLAR REGION ACCORDING TO MODIFIED MODEL

Figure 2. Lines of Equal Exospheric Temperature in the Polar Region According to Modification of Jacchia's Model Suggested in this Paper